الجامعة السورية الخاصة
SYRIAN PRIVATE UNIVERSITY

كـلـيــة هندسة الحاسوب والمعلوماتية
Computer and Informatics Engineering Faculty

Electric Circuits I

Dr. Eng.
Hassan M. Ahmad
Hassan.Ahmad@spu.edu.sy, istamo48@mail.ru

Chapter 2
Basic Laws

2.1 Ohm's Law.

2.2 Nodes, Branches, and Loops.

2.3 Kirchhoff's Laws.

2.4 Series Resistors and Voltage Division.

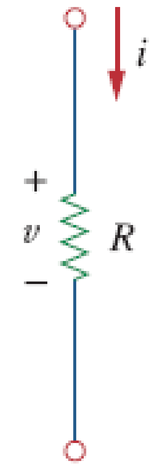
2.5 Parallel Resistors and Current Division.

2.6 Wye-Delta Transformations.

2.1 Ohms Law

- **Ohm's law** states that the voltage across a resistor is directly proportional to the current ***i*** flowing through the resistor *R*.
- Mathematical expression for Ohm's Law is as follows:

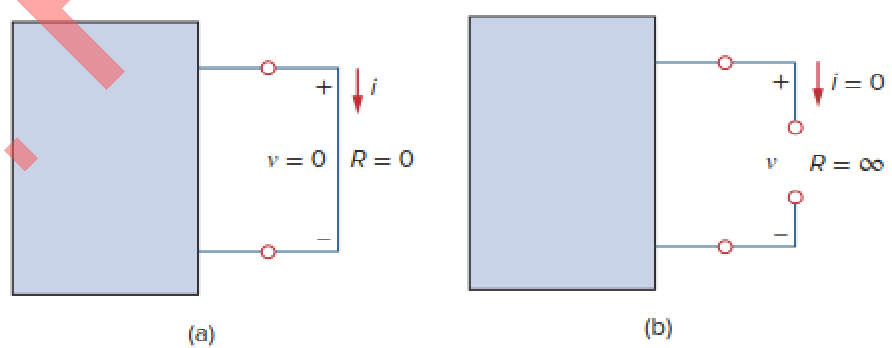
$$v = iR$$



- The resistance *R* of an element denotes its *ability* to resist the flow of electric current; it is measured in ohms (**Ω**).
- Two extreme possible values of R: **0 (zero)** **and ∞ (infinite)** are related with two basic circuit concepts:

a) short circuit;

b) open circuit.

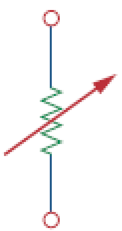


Types of resistors

- **Fixed resistor**: its resistance remains constant.

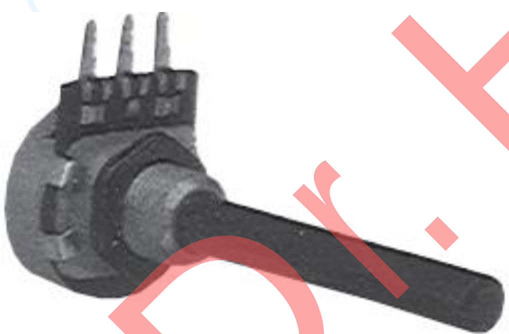
❖ There are two common types of fixed resistors:
  - Wirewound, Fig. (a);
  - Composition, Fig. (b), used when large resistance is needed.

- **Variable resistor**: its resistance is variable.
  - Wirewound, Fig. (c);
  - Composition, Fig. (d).



(a)

(b)



(c)

(d)

- **Conductance** is the ability of an element to conduct electric current; it is the reciprocal of resistance R and is measured in siemens (S).

$$G = \frac{1}{R} = \frac{i}{v}$$

- **Power dissipated** by a resistor: $p = vi = i^2 R = \frac{v^2}{R} = v^2 G = \frac{i^2}{G}$

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Example 2.1.

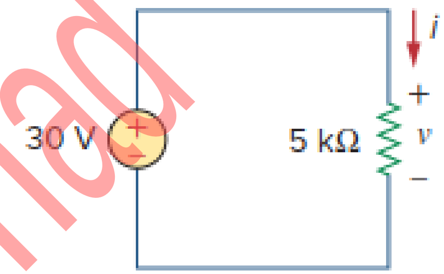
An electric iron draws 2 A at 120 V. Find its resistance.

Solution:

$$R = \frac{v}{i} = \frac{120}{2} = 60\,\Omega$$

Example 2.2

In the circuit shown in Fig., calculate *i*, *G*, and *p*.



Solution:

$$i = \frac{v}{R} = \frac{30}{5\times10^{3}} = 6\ \text{mA}; \qquad G = \frac{1}{R} = \frac{1}{5\times10^{3}} = 0.2\ \text{mS}$$

We can calculate the power in various ways:

$$p = vi = 30(6\times10^{-3}) = 180\ \text{mW}; \text{ or } \ p = i^{2}R = (6\times10^{-3})5\times10^{3} = 180\ \text{mW}$$

$$p = v^{2}G = (30)^{2}0.2\times10^{-3} = 180\ \text{mW}$$

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Example 2.3

A voltage source of 20sinπt V is connected across a 5-kΩ resistor. Find the current through the resistor and the power dissipated.

Solution:

$$i = \frac{v}{R} = \frac{20\sin\pi t}{5\times10^{-3}} = 4\sin\pi t\ \text{mA}$$

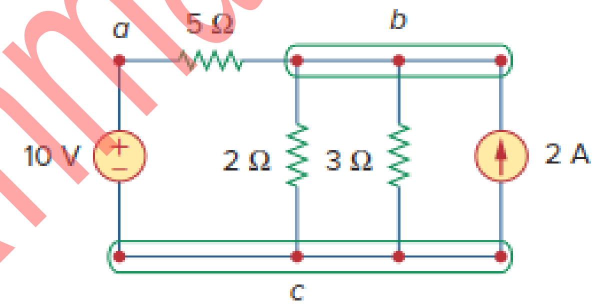
$$p = vi = 80\sin^{2}\pi t\ \text{mW}$$

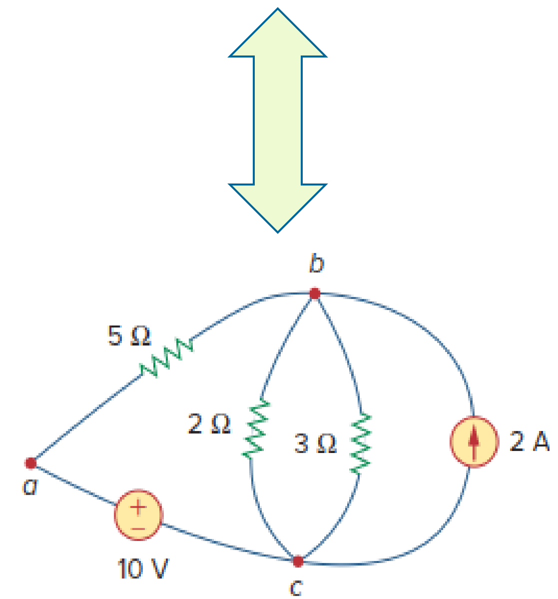
2.2 Nodes, Branches and Loops

- A **branch** represents a single element such as a voltage source or a resistor.
- A **node** is the point of connection between two or more branches.
- A **loop** is any closed path in a circuit.
- A network with *b* branches, *n* nodes, and *l* independent loops will satisfy the fundamental theorem of **network topology**:

$$b = l + n - 1$$

- Two or more elements are in **series** if they exclusively share a single node and consequently carry the same current.
- Two or more elements are in **parallel** if they are connected to the same two nodes and consequently have the same voltage across them.



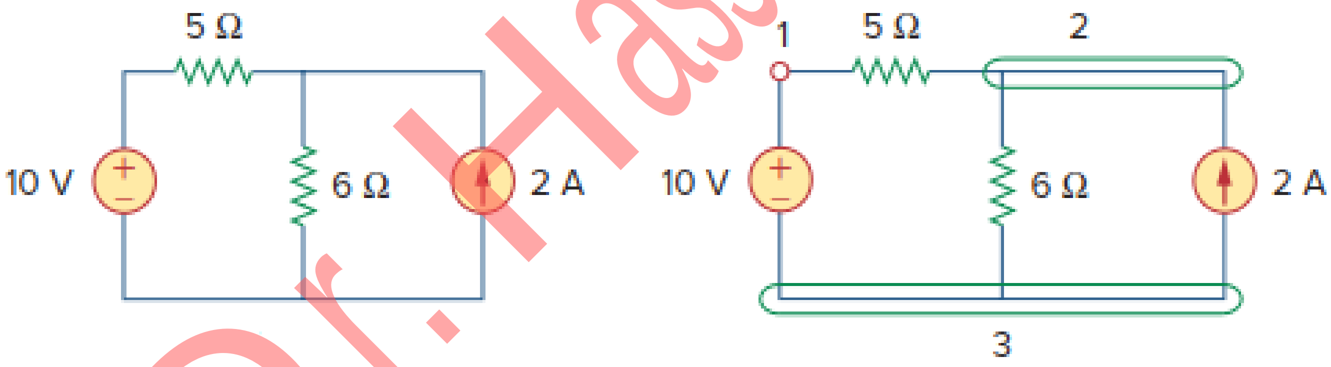


Example 2.3.

Determine the number of branches and nodes in the circuit shown in Fig. Identify which elements are in series and which are in parallel.

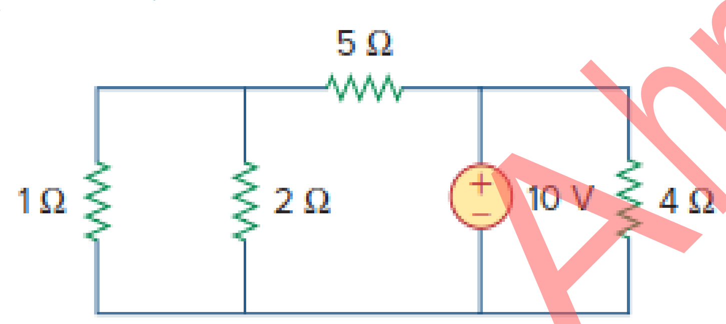
Solution:

- Since there are four elements in the circuit, the circuit has four branches: 10 V, 5Ω , 6Ω and 2 A.
- The circuit has three nodes as identified in Fig.
- The 5-Ω resistor is in series with the 10-V voltage source because the same current would flow in both.
- The 6-Ω resistor is in parallel with the 2-A current source because both are connected to the same nodes 2 and 3.



Example 2.4

How many branches and nodes does the circuit in Fig. have? Identify the elements that are in series and in parallel.

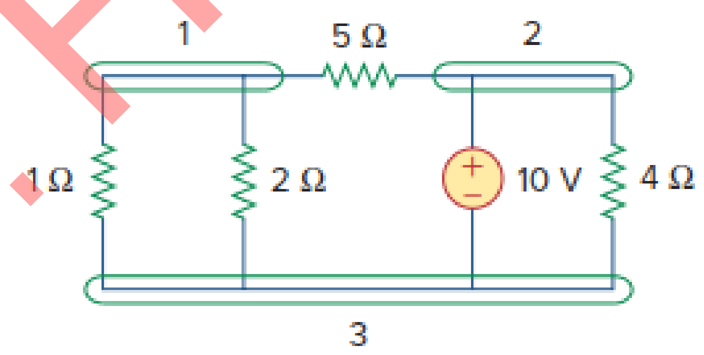


***Solution*:**

Five branches and three nodes are identified in Fig.

The 1-Ω and 2-Ω resistors are in parallel.

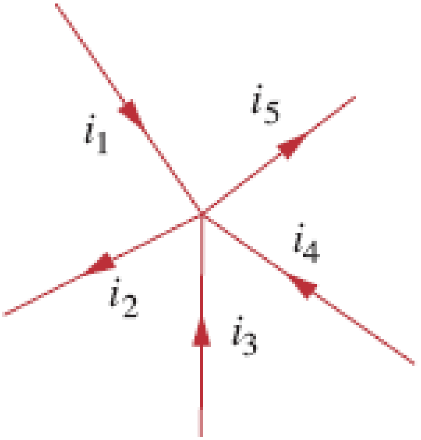
The 4-Ω resistor and 10-V source are also in parallel.



2.3 Kirchhoff's Laws

- **Kirchhoff's current law** (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.
- Mathematically, $\sum_{n=1}^{N} i_n = 0 \;\Rightarrow\; i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$
- In other words, KCL states that the sum of the currents entering a node is equal to the sum of the currents leaving the node.

$$i_1 + i_3 + i_4 = i_2 + i_5$$

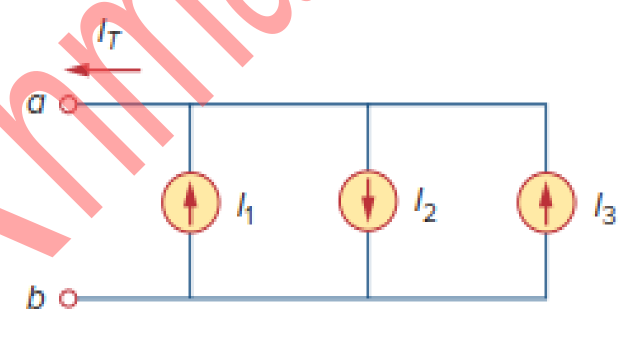


Application of KCL

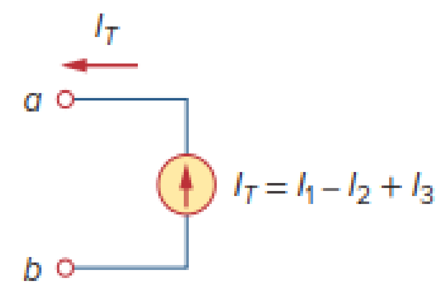
- A simple *application* of KCL is **combining current sources in parallel**.
- The combined current is the algebraic sum of the current supplied by the individual sources.
- For example, the current sources shown in Fig. (a) can be combined as in Fig. (b).
- Applying KCL to node *a:*

$$I_T + I_2 = I_1 + I_3 \Rightarrow I_T = I_1 - I_2 + I_3$$

- A circuit cannot contain two different currents, and , in series, unless $I_1 = I_2$; otherwise KCL will be violated.



(a)

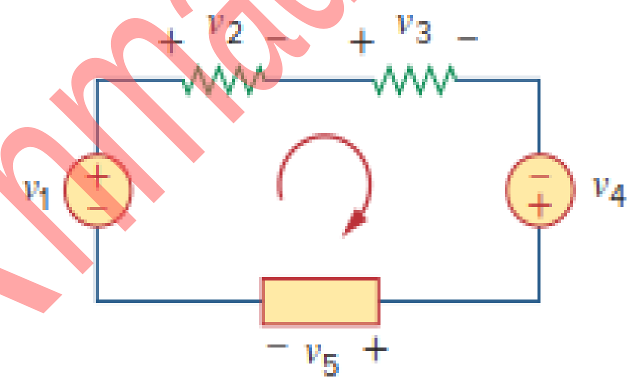


(b)

- **Kirchhoff's voltage law** (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.
- Mathematically,

$$\sum_{m=1}^{M} v_n = 0$$

- Consider the circuit in Fig.



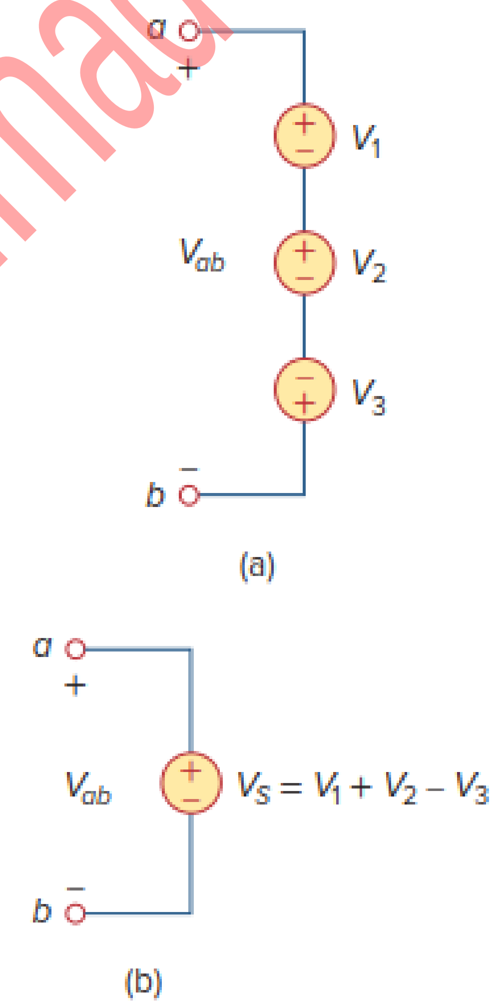
  - Suppose we start with the voltage source and go clockwise around the loop as shown; then voltages would be $-v_1$, $+v_2$, $+v_3$, $-v_4$, and $+v_5$ in that order.
  - Thus, KVL yields $-v_1 + v_2 + v_3 - v_4 + v_5 = 0 \Leftrightarrow v_2 + v_3 + v_5 = v_1 + v_4$

Application of KVL

- When voltage sources are connected in series, KVL can be applied to obtain the total voltage.
- Consider the voltage sources shown in Fig.(a).
- Applying KVL gives the combined or equivalent voltage source in Fig.(b).

$$-V_{ab} + V_1 + V_2 - V_3 = 0 \Rightarrow V_{ab} = V_1 + V_2 - V_3$$

- To avoid violating KVL, a circuit cannot contain two different voltages V_1 and V_2 in parallel unless $V_1 = V_2$.



Example 2.5

For the circuit in Fig.(a), find voltages v_1 and v_2.

***Solution*:**

- To find v_1 and v_2 we apply Ohm's law and Kirchhoff's voltage law.
- Assume that current i flows through the loop as shown in Fig.(b).
- From Ohm's law, $v_1 = 2i, \quad v_2 = -3i \qquad (1)$
- Applying KVL around the loop gives

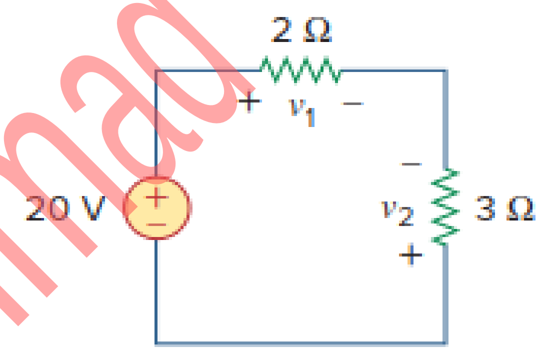
$$-20 + v_1 - v_2 = 0 \qquad (2)$$

- By Substituting Eq.(1) into Eq.(2):

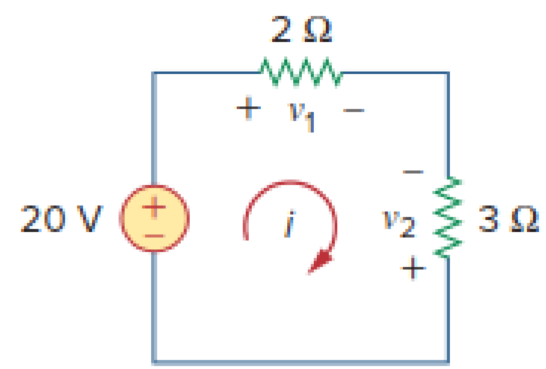
$$-20 + 2i + 3i = 0 \Rightarrow i = 4 \text{ A} \qquad (3)$$

- By Substituting Eq.(3) into Eq.(1):

$$v_1 = 8\,\text{V}, \quad v_2 = -12 \text{ V}$$



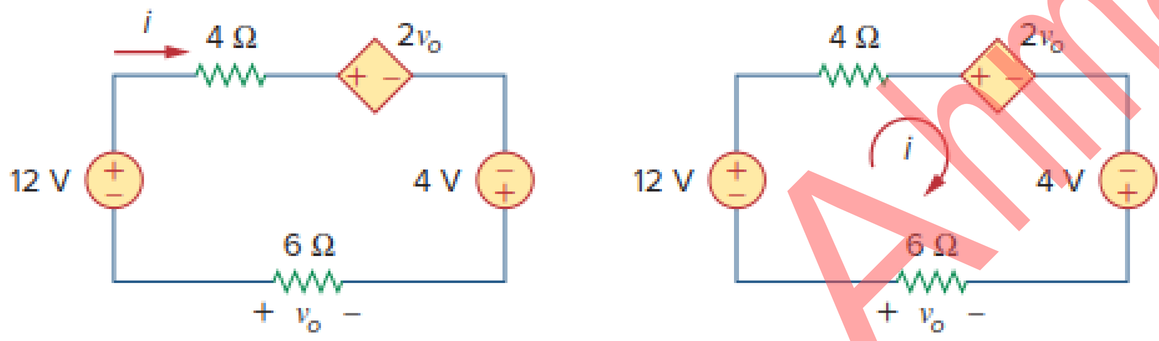
(a)



(b)

Example 2.6

Determine v_o and i in the circuit shown in Fig.(a).



(a) (b)

Solution:

- We apply KVL around the loop as shown in Fig. (b).

$$-12 + 4i + 2v_o - 4 + 6i = 0 \qquad (1)$$

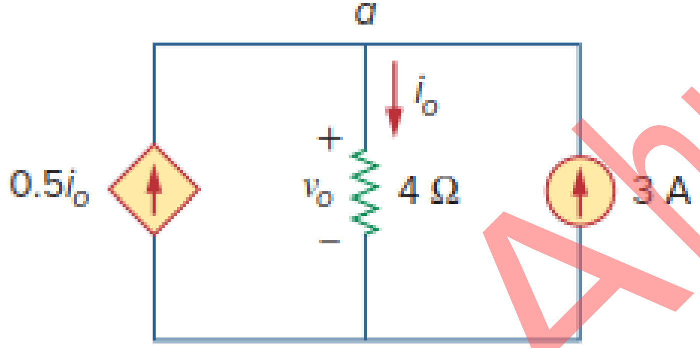
- Applying Ohm's law to the 6-Ω resistor gives: $v_o = -6i \qquad (2)$

- Substituting Eq. (2) into Eq. (1): $-16 + 10i - 12i = 0 \Rightarrow i = -8\,\text{A}$

- Then, $v_o = -6 \times (-8) = 48\,\text{V}$

Example 2.7

Find current i_o and voltage v_o in the circuit shown in Fig.



Solution:

- Applying KCL to node *a*, we obtain $3+0.5i_o = i_o \Rightarrow i_o = 6\,\text{A}$
- For the 4-Ω resistor, Ohm's law gives $v_o = 4i_o = 24\text{ V}$

Example 2.8

Find currents and voltages in the circuit shown in Fig.(a).

Solution:

- By Ohm's law,

$$v_1 = 8i_1, \quad v_2 = 3i_2, \quad v_3 = 6i_3 \qquad (1)$$

- At node *a*, KCL gives $i_1 - i_2 - i_3 = 0 \qquad (2)$
- Applying KVL to loop 1 as in Fig. (b),

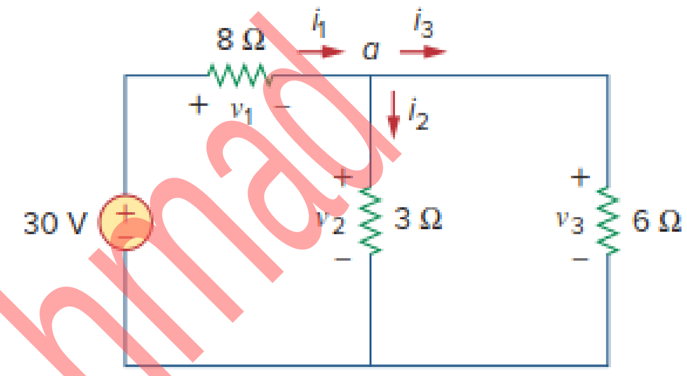
$$-30 + v_1 + v_2 = 0 \qquad (3)$$

- Substituting from Eq. (1) into Eq. (3):

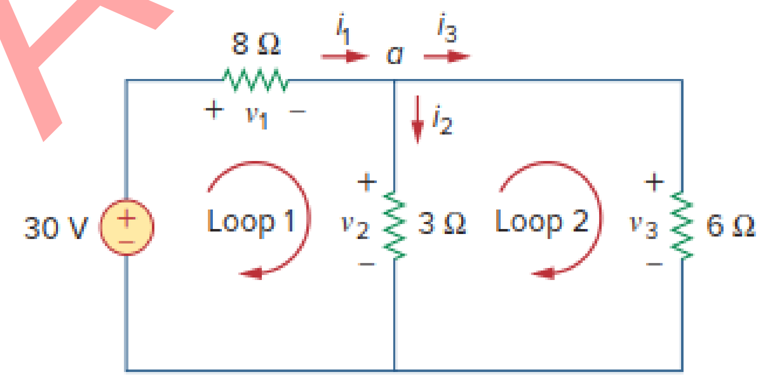
$$-30 + 8i_1 + 3i_2 = 0 \Rightarrow i_1 = \frac{30 - 3i_2}{8} \qquad (4)$$

- Applying KVL to loop 2, $-v_2 + v_3 = 0 \Rightarrow v_3 = v_2$ or $6i_3 = 3i_2 \Rightarrow i_3 = \frac{i_2}{2} \qquad (5)$
- Substituting Eqs. (4) and (5) into (2) gives $\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0 \Rightarrow i_2 = 2\,\text{A}$
- Then,

$$i_1 = 3\,\text{A},\ i_3 = 1\,\text{A},\ v_1 = 24\,\text{V},\ v_2 = 6\,\text{V},\ v_3 = 6\,\text{V}$$



(a)

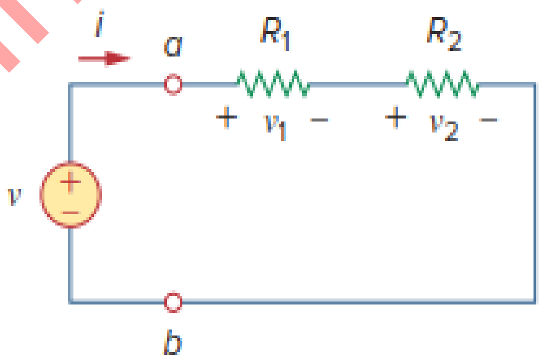


(b)

2.4 Series Resistors and Voltage Division

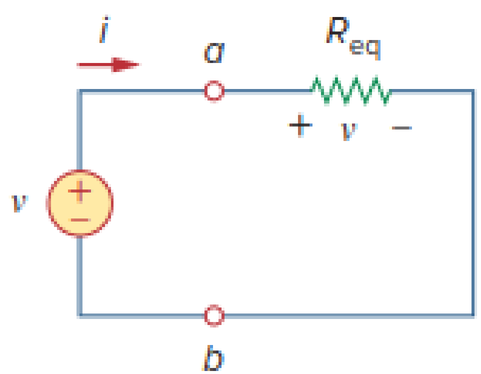
- **Series**: two or more elements are in series if they are cascaded or connected sequentially and consequently carry the same current.
- The **equivalent resistance** of any number of resistors connected in a series is the sum of the individual resistances.

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$



- The **voltage divider rule (VDR)** can be expressed as

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

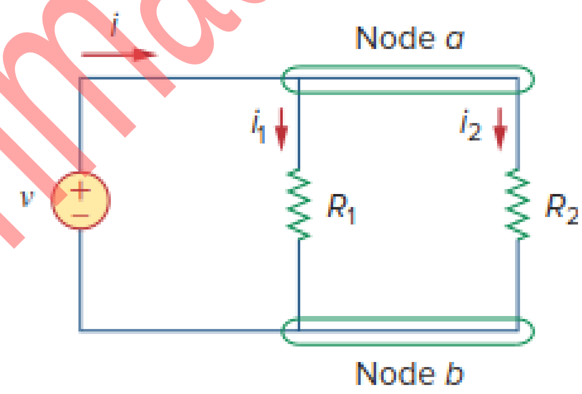


- Applying VDR to given circuit gives

$$v_1 = \frac{R_1}{R_1 + R_2} v; \quad v_2 = \frac{R_2}{R_1 + R_2} v$$

2.5 Parallel Resistors and Current Division

- **Parallel**: Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.
- The ***equivalent resistance*** of a circuit with N resistors in *parallel* is:



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

- For given circuit: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

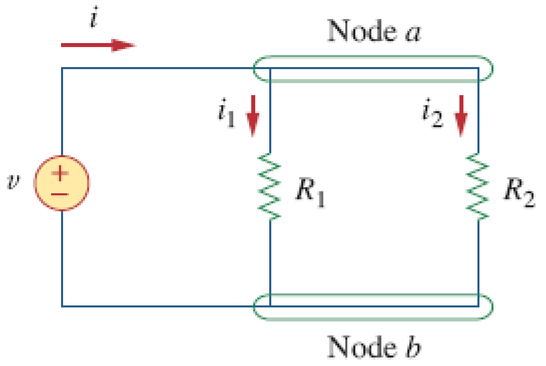
- The **total current *i*** is shared by the resistors in inverse proportion to their resistances.

$$i = i_1 + i_2 = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = v\frac{1}{R_{eq}}$$

- The **equivalent conductance** of resistors connected in parallel is the sum of their individual conductance.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \Leftrightarrow G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

- Given the total current i entering node a in Fig., how do we obtain current i_1 and i_2?
- The **current divider rule (CDR)**.



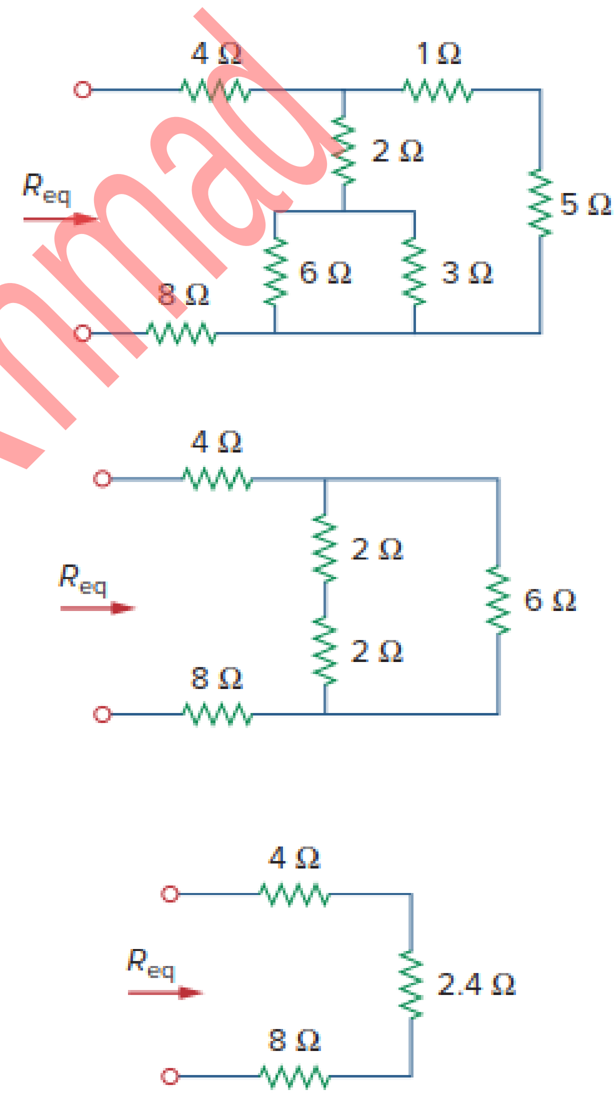
$$v = iR_{eq} = i\frac{R_1R_2}{R_1 + R_2}; \quad i_1 = \frac{v}{R_1}; \quad i_2 = \frac{v}{R_2}$$

$$\Rightarrow i_1 = \frac{R_2}{R_1 + R_2} i \quad \text{and} \quad i_2 = \frac{R_1}{R_1 + R_2} i$$

$$i_1 = \frac{G_1}{G_1 + G_2} i, \quad i_2 = \frac{G_2}{G_1 + G_2} i \Rightarrow i_n = \frac{G_n}{G_1 + G_2 + \cdots + G_N} i$$

Example 2.9

Find R_{eq} for the circuit shown in Fig.



Solution:

$$6\Omega \parallel 3\Omega = \frac{6\times 3}{6+3} = 2\Omega$$

$$1\Omega + 5\Omega = 6\Omega$$

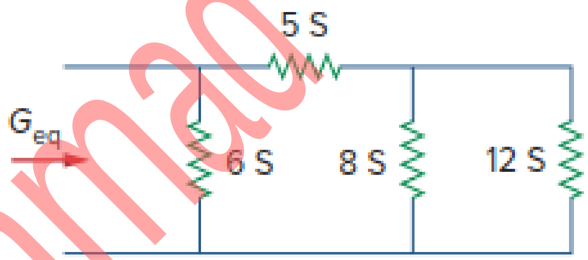
$$2\Omega + 2\Omega = 4\Omega$$

$$4\Omega \parallel 6\Omega = \frac{4\times 6}{4+6} = 2.4\Omega$$

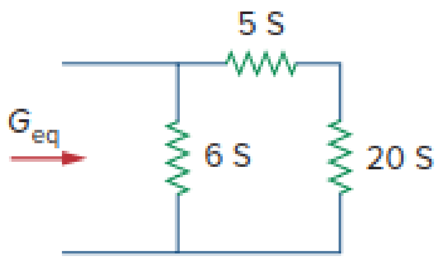
$$R_{eq} = 4\Omega + 2.4\Omega + 8\Omega = 14.4\Omega$$

Example 2.10

Find the equivalent conductance G_{eq} for the circuit in Fig.(a).



(a)



(b)

Solution:

- The 8-S and 12-S resistors are in parallel, so

$$8S + 12S = 20S$$

- This 20-S resistor is now in series with 5 S as shown in Fig. (b), so

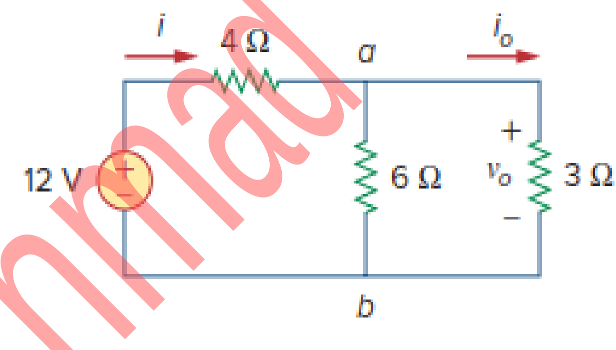
$$\frac{20 \times 5}{20 + 5} = 4S$$

- This 4-S is in parallel with the 6-S resistor. Hence,

$$G_{eq} = 6 + 4 = 10S$$

Example 2.11

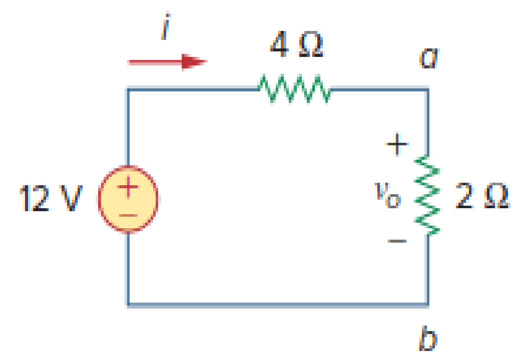
Find i_o and v_o in the circuit shown in Fig.(a). Calculate the power dissipated in the 3-Ω resistor.



(a)

Solution:

$$6\Omega \,||\, 3\Omega = \frac{6\times 3}{6+3} = 2\Omega$$



(b)

Because the resistors 6-Ω and 3-Ω are in parallel, therefore they have the same voltage v_o.

So, we can obtain v_o in two ways.

One way is to apply Ohm's law to get:

$$i = \frac{12}{4+2} = 2\,A \Rightarrow v_o = 2i = 2\times 2 = 4\text{ V}$$

$$\text{and,}\quad v_o = 3i_o = 4 \Rightarrow i_o = \frac{4}{3}\text{A}$$

Another way is to apply voltage division (VDR) and current division (CDR) to the circuit, then

$$v_o = \frac{2}{2+4}(12\text{ V}) = 4\text{ V},\qquad i_o = \frac{6}{6+3}i = \frac{2}{3}(2\text{ A}) = \frac{4}{3}\text{A}$$

The power dissipated in the 3-Ω resistor is $p_o = v_o i_o = 4\left(\frac{4}{3}\right) = 5.333\text{ W}$

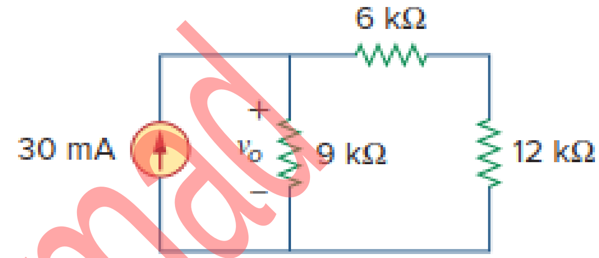
Example 2.12

For the circuit shown in Fig.(a), determine:

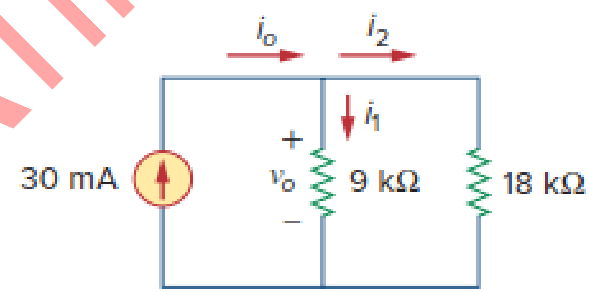
a) the voltage v_o,

b) the power supplied by the current source,

c) the power absorbed by each resistor.



(a)



(b)

Solution:

The 6-k Ω and 12-k Ω in series ➔ 6 + 12 = 18 kΩ.

Thus ➔ Fig.(b). Now, apply the current division technique to find i_1 and i_2.

$$i_1 = \frac{18\times10^3}{(9+18)\times10^3}(30\,\text{mA}) = 20\,\text{mA}, \quad i_2 = \frac{9\times10^3}{(9+18)\times10^3}(30\,\text{mA}) = 10\,\text{mA}$$

Notice that the voltage across the 9-kΩ and 18-kΩ resistors is the same, and $\mathbf{v_o}$ = 9,000i_1 = 18,000i_2 = 180 V, as expected.

Power supplied by the source is $p_o = v_o i_o = 180V \times 30\,\text{mA} = 5.4\,\text{W}$

Power absorbed by the 12-kΩ resistor is

$$p = iv = i_2(i_2 R) = i_2^2 R = (10\times10^{-3})^2(12\times10^3) = 1.2\,\text{W}$$

Power absorbed by the 6-kΩ resistor is $p = i_2^2 R = (10\times10^{-3})^2(6\times10^3) = 0.6\,\text{W}$

Power absorbed by the 9-kΩ resistor is

$$p = i_1 v_o = \frac{v_o}{R} v_o = \frac{v_o^2}{R} = (10\times10^{-3})^2(6\times10^3) = 3.6W$$

Power supplied = the power absorbed = 3.6 W

2.6 Wye-Delta Transformations

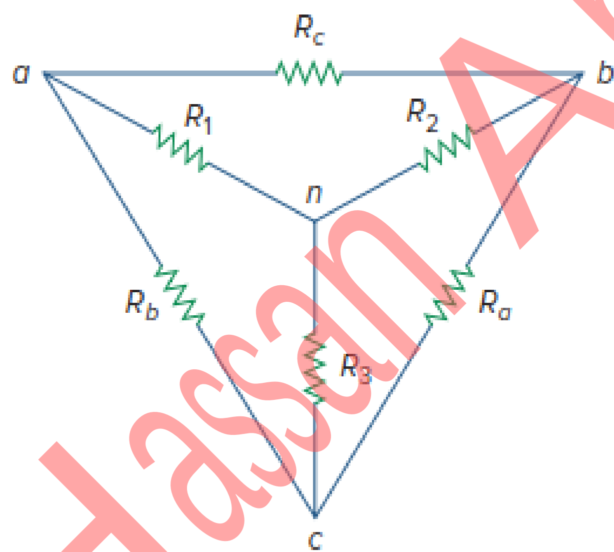
Delta to Wye (Star) Conversion

$\Delta \rightarrow \mathrm{Y}$

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



Wye (Star) to Delta Conversion

$\mathrm{Y} \rightarrow \Delta$

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

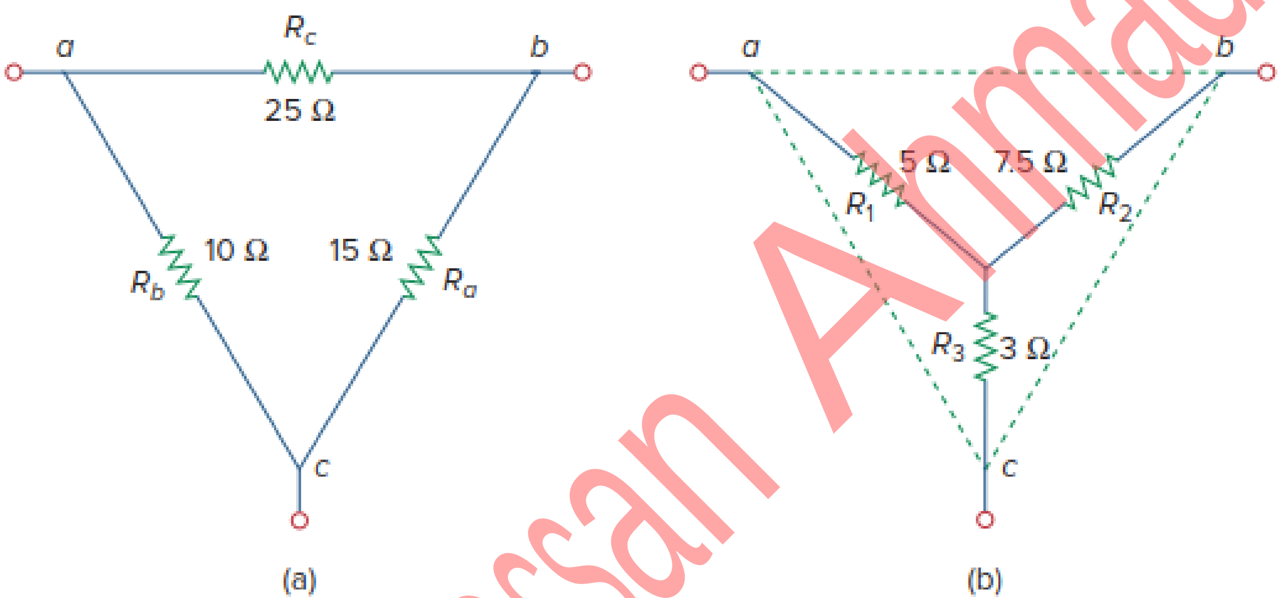
The Y and Δ networks are said to be balanced when

$$R_1 = R_2 = R_3 = R_\mathrm{Y}, \qquad R_a = R_b = R_c = R_\Delta$$

$$R_\mathrm{Y} = \frac{R_\Delta}{3} \quad \text{or} \quad R_\Delta = 3R_\mathrm{Y}$$

Example 2.13

Convert the Δ network in Fig. (a) to an equivalent Y network.



(a) (b)

Solution:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = 5\Omega$$

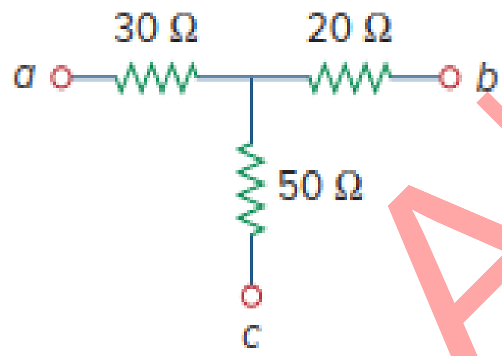
$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{15 + 10 + 25} = 7.5\Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{15 + 10 + 25} = 3\Omega$$

The equivalent Y network is shown in Fig.(b).

Example 2.14

Convert the Y network in Fig. to an equivalent Δ network.

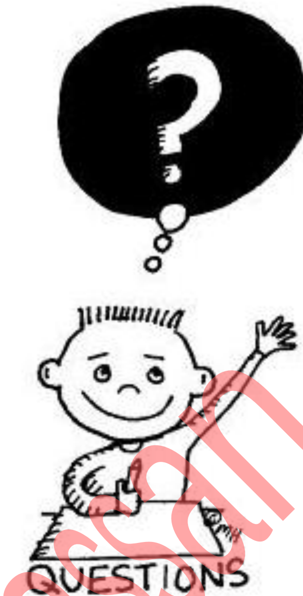


Solution:

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{30\times 20 + 30\times 50 + 20\times 50}{30} = 103.3\Omega$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{30\times 20 + 30\times 50 + 20\times 50}{20} = 155\Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{30\times 20 + 30\times 50 + 20\times 50}{50} = 62\Omega$$



The end of chapter 2